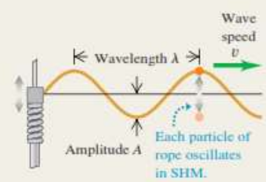
Seminary 7
MECHANICAL WAVES

SUMMARY (see course):

Waves and their properties: A wave is any disturbance that propagates from one region to another. A mechanical wave travels within some material called the medium. The wave speed v depends on the type of wave and the properties of the medium.

In a periodic wave, the motion of each point of the medium is periodic with frequency f and period T. The wavelength λ is the distance over which the wave pattern repeats, and the amplitude A is the maximum displacement of a particle in the medium. The product of λ and f equals the wave speed. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion. (See Example 15.1.)

$$v = \lambda f \qquad (15.1)$$



Wave functions and wave dynamics: The wave function $y(x, t)$ describes the displacements of individual particles in the medium. Equations (15.3), (15.4), and (15.7) give the wave equation for a sinusoidal wave traveling in the $+x$-direction. If the wave is moving in the $-x$-direction, the minus signs in the cosine functions are replaced by plus signs. (See Example 15.2.)

The wave function obeys a partial differential equation called the wave equation, Eq. (15.12).

The speed of transverse waves on a string depends on the tension F and mass per unit length μ. (See Example 15.3.)

$$y(x, t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right] = A\cos 2\pi f\left(\frac{x}{v} - t\right) \qquad (15.3)$$

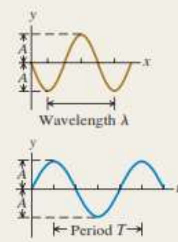
$$y(x, t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \qquad (15.4)$$

$$y(x, t) = A\cos(kx - \omega t) \qquad (15.7)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f = vk$

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x, t)}{\partial t^2} \qquad (15.12)$$

$$v = \sqrt{\frac{F}{\mu}} \quad \text{(waves on a string)} \qquad (15.13)$$



Wave power: Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power P_{av} is proportional to the square of the wave amplitude and the square of the frequency. For waves that spread out in three dimensions, the wave intensity I is inversely proportional to the square of the distance from the source. (See Examples 15.4 and 15.5.)

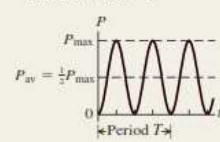
$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 \qquad (15.25)$$

(average power, sinusoidal wave)

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \qquad (15.26)$$

(inverse-square law for intensity)

Wave power versus time t at coordinate $x = 0$



Extracted from:
SEARS AND ZEMANSKY'S UNIVERSITY PHYSICS, WITH MODERN PHYSICS 13TH EDITION.

PERIODIC WAVES

1/ A certain transverse wave is described by:

$$y(x,t) = (6.50mm)\cos 2\pi\left(\frac{x}{28.0cm} - \frac{t}{0.0360s}\right)$$

Determine the wave's (a) Amplitude; (b) wavelength (c) frequency (d) speed of propagation (e) direction of propagation.

2/ (a) Show that the wave equation: $y(x,t) = A\cos(kx - \omega t)$ may be written as:

$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$ where *v* is the wave speed of propagation

(b) Use *y(x,t)* to find an expression for the transverse velocity $v_y(x,t)$ of a particle in the string on which the wave travels.

(c) Find the maximum speed of the particle in the string. Under what circumstances is this equal to the propagation speed v? Less than v? Greater than v?

3/ Wave equation is a linear equation.

Let $y_1(x,t) = A\cos(k_1 x - \omega_1 t)$ and $y_2(x,t) = A\cos(k_2 x - \omega_2 t)$ be solutions of the wave equation for the same v. Show that $y(x,t) = y_1(x,t) + y_2(x,t)$ is also a solution of the wave equation.

4/ A rope of mass 0.65 kg is stretched between two supports, 30 m apart. If the tension in the rope is 120 N, how long will it take a pulse to travel from one support to the other?

5/ An earthquake generates three kinds of waves: surface waves (L-waves), which are the slowest and weakest, shear (S) waves, which are transverse waves and carry most of the energy, and pressure (P) waves, which are longitudinal waves and are the fastest. The speed of P waves is approximately 7 km/s, and that of S waves is about 4 km/s. People do not generally feel the P waves, but animals seem to be sensitive to them.

If a person reports that her dog started barking 20 seconds "before the earthquake," then approximately how far was the origin of the earthquake?

ENERGY IN WAVE MOTION

1/ A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

2/ Energy Output. By measurement you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is 0.026 W/m2 at a distance of 4.3 m from the source. (a) What is the intensity at a distance of 3.1 m from the source? (b) How much sound energy does the source emit in one hour if its power output remains constant?

3/ A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is:

$$y(x,t) = 2.3\text{mm} \cos[(6.98\text{rad/m})x + (742\text{rad/s})t]$$

Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.00338 kg. You are then asked to determine the following: (a) amplitude; (b) frequency; (c) wavelength; (d) wave speed; (e) direction the wave is traveling; (f) tension in the rope; (g) average power transmitted by the wave.

4/ At a distance of 7×10^{12} m from a star, the intensity of the radiation from the star is 15.4 W/m^2 . Assuming that the star radiates uniformly in all directions, what is the total power output of the star?

QUESTIONS:

1/ The amplitude of a wave decreases gradually as the wave travels down a long, stretched string. What happens to the energy of the wave when this happens?

2/ A sinusoidal wave can be described by a cosine function, which is negative just as often as positive. So why isn't the average power delivered by this wave zero?

3/ If you are walking on the moon, can you hear cracking sound behind you? Explain.

4/ Why does the flash of light reach the earth before than the sound coming from the same thunder? Explain.

5/ What is the relationship between frequency and wavelength?